The Complete A-Level Guide to Logarithms

General Formula:

Any number, n, can be expressed as a power, p, of a base, b, thusly:

$n = b^p$

where $b = \sqrt[p]{n}$

and $p = \log_b(n)$

The logarithmic function, $\log_b(n) = p$, is the mathematically inverse function to exponetiation.

Identities:

The logarithmic function has a few established identities as follows:

Product	$\log_b(xy) = \log_b(x) + \log_b(y)$
Quotient	$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
Power	$\log_b(x^p) = p\log_b(x)$
Root	$\log_b(\sqrt[p]{x}) = \frac{\log_b(x)}{p}$
Change of Base	$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$

Particular Bases:

The logarithmic function is often used with particular bases which have specialised names and notation as follows:

Base b	Name	Notation
2	Binary Logarithm	$\mathrm{lb}(x)$
e	Natural Logarithm	$\ln(x)$

10	Common Logarithm	$\lg(x), \log(x)$
b	Logarithm to Base b	$\log_b(x)$

Examples of Logarithmic Scales and their Related Expressions:

Logarithmic scales are used within many contexts and logarithmic equations are used to calculate many quantities, below are just a few:

Richter Scale	$M = \lg\left(\frac{I}{S}\right)$
pH Scale	$pH\ of\ solution = -\lg(Concentration\ of\ H^+\ ions)$
Decibel Scale	$dB = 10\lg\left(\frac{S_1}{S_2}\right)$
Half-Life	$t_{1/2} = \frac{t}{\mathrm{lb}\left(\frac{N_O}{N(t)}\right)}$